AE 3610, Lab #03

Buckle-Up for Science: An Exploration of Load vs. Displacement-based Strain and Buckling Stress

Or

I’m Ready for Mohr: Finally Finding a Use for Mohr’s Circle

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Introduction

We must understand the effects of bending and buckling in order to create safe, predictable, and durable machines for our field. Team A11 conducted laboratory experiments exploring cantilever beam theory and the theory of buckling. For the exploration of cantilever beam theory, we used strain gauges attached to a thin metal rod specimen rigged to a stationary table. In order to measure strain cause by loads, we attached weights to one specified point along the beam. In order to measure strain caused by displacement, we used a micrometer to cause a measured displacement on another specified point along the beam. For the exploration of buckling, we placed strain gauges at the midpoint and on either side of both a short and long Aluminum column, then placed the columns in the Instron machine and induced a compressive force.

Data Results

Raw Data

<u>Load-Controlled Cantilever Beam Experiment</u>

We conducted the load-controlled portion of the cantilever beam experiment by placing our 6 (six) predetermined loads onto the beam one at a time (in some cases daisy-chaining the weights in order to make up the desired load). After watching the reports stabilize on the computer software, a team member recorded each of the three strain gauge readings. Table I shows the three strain gauge readings from the 6 loads as well as the drift effect at zero load after the last load (1000g) was removed.

<u>Displacement-Controlled Cantilever Beam Experiment</u>

The displacement-controlled cantilever beam experiment was the same set-up as the load-controlled cantilever beam experiment, except that the displacement measured by the strain gauge on the beam specimen was enforced by a micrometer. The six predetermined displacements were enforced one at a time and recorded one at a time, just as with the previous experiment. Table II shows the strain gauge readings from the 6 displacements as well as the drift effect at zero displacement after the last displacement was removed.

Column Buckling Experiments

The team connected the strain gauges from each of the long and short columns to sensors, which we then monitored as the compressive load from the Instron was applied. A team member recorded videos of the strain monitors as the load on each the long and short columns was steadily increased and then decreased. Tables IV and V show the strain gauge data taken from the video recordings for the long and short columns respectively.

Reduced Data

Load-Controlled Cantilever Beam Experiment

The strain gauge attached to the beam specimen measured strain in three directions. Those three directions were unknown, but were related to the principal strain the cantilever experienced through the following method:

$$\varepsilon_X^* = \varepsilon_A \qquad (1)$$

$$\varepsilon_Y^* = \varepsilon_C \qquad (2)$$

$$\frac{\gamma_{XY}^*}{2} = \varepsilon_B - \frac{\varepsilon_A + \varepsilon_C}{2} \qquad (3)$$

ε_A, ε_B, and ε_C were the strains measured by the three strain gauges. The starred values are arbitrary values.

First, equations (1) – (3) were used to define a set of strains rotated by some angle about the same axis (the z-axis as defined in this problem) (see Table I and Figure 1). Because all shear strains were assumed to be zero due to the nature of the problem, we could find the principal strains using equations (4) and (5).

$$\varepsilon_1 = \frac{\varepsilon_X^* + \varepsilon_Y^*}{2} + \left[\left(\frac{\varepsilon_X^* - \varepsilon_Y^*}{2}\right)^2 + \left(\frac{\gamma_{XY}^*}{2}\right)^2\right]^{1/2} \quad (4)$$

$$\varepsilon_2 = \frac{\varepsilon_X^* + \varepsilon_Y^*}{2} - \left[\left(\frac{\varepsilon_X^* - \varepsilon_Y^*}{2}\right)^2 + \left(\frac{\gamma_{XY}^*}{2}\right)^2\right]^{1/2} \quad (5)$$

ε_1 and ε_2 are the principal strains due to the loads applied on the beam. Figure 3 shows the principal strain ε_1 and its linear relation to the applied load P.

$$\sigma_X = \frac{My}{I} \quad (6)$$

σ_x is the stress in the x-direction (along the beam), M is the bending moment at the strain gauge's location, y is the neutral axis of the specimen (the thickness divided by two), and I is the moment of inertia.

To calculate the elastic modulus, E, the slope from the best fit line in Figure 3 was inverted to be force/strain, and was then multiplied by the distance between the strain gauge and the load point (making M for the numerator). Then it was multiplied by the position of the neutral axis (y), and then divided by the moment of inertia (I), creating the equation for stress divided by strain, which is the exact determination of the elastic modulus, E. E was calculated as **42.2 GPa** (as reported in Table III).

<u>Displacement-Controlled Cantilever Beam Experiment</u>

The displacement-controlled experiment used the same methods as the load-controlled experiment and therefore the data reduced in much the same way: an arbitrary set of strain axes was created using equations (1) – (3) (see Table II and Figure 2), and then principal strains were calculated using equations (4) and (5). These were then plotted (see Figure 4) and a best-fit line was found in Excel.

Equation (6) was used again as well as the calculated elastic modulus of 42.2 GPa from the previous experiment: the slope from the best-fit line (strain/displacement) was multiplied by the elastic modulus (E) and the moment of inertia (I), and then divided by the distance between the strain gauge and load point (d) and the neutral axis point (y) in order for the dimensional analysis to return Newtons per meter, which is the measure of effective bending spring stiffness, K.

K was calculated to be **861.2 N/m** (as reported in Table III).

Column Buckling Experiments

Strains measured in the column buckling experiments were converted into Southwell plots (Figures 5 and 6). The x-axes of the plots are the measured strain values at the pre-determined load points divided by the load itself. The slope of each of the Southwell plots represents the experimentally-devised critical load for each beam (Table VI). For the long column, the critical load P_{cr} was determined to be **4340 N** while the critical load for the short column was determined to be **5840 N.**

For comparison, the theoretical values for the critical loading for each column were also calculated (Table VI) using equation (7):

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (7)$$

E is the elastic modulus calculated from the earlier experimental segment, I is the moment of inertia, and L is the length of the column. The theoretical loads were calculated as **2494 N** and **3642 N** for the long and short columns respectively.

Discussion

Supplemental Questions

<u>Bending</u>

In considering a cantilevered, rectangular beam subjected to a load produced by a 1 kg point mass at the (free) tip of beam and assuming that the beam material is made of Aluminum and that the beam is 11 inches long, 1 inch wide, and 1/8th inch thick:

1. Based on the Euler-Bernoulli beam theory, the deflection (in millimeters) of the beam is:

$$m = 1kg$$

$$P = m \cdot g = 1kg \cdot 9.81 \frac{m}{s^2} = 9.81\ N$$

$$I = \frac{w \cdot h^3}{12} = \frac{1in \cdot \left(\frac{1}{8} in\right)^3}{12} \cdot \frac{0.0254^4 m^4}{in^4} = 6.775 \times 10^{-11} m^4$$

$$\delta = \frac{PL^3}{3EI} = \frac{(9.81\ N)\left(11\ in \cdot \frac{0.0254 in}{m}\right)^3}{3(70 \times 10^9 Pa)(6.775 \times 10^{-11} m^4)} \cdot 10^3 = \mathbf{15\ mm}$$

2. The longitudinal stress (in MPa), longitudinal strain, and transverse strain at the root of the beam are:

$$y = \frac{h}{2} = \frac{\frac{1}{8} in}{2} \cdot \frac{0.0254\ m}{in} = 0.02286\ m$$

$$\sigma_x = \frac{My}{I} = \frac{(9.81N)(11\ in)\left(\frac{0.0254\ m}{in}\right)(0.02286\ m)}{6.775 \times 10^{-11} m^4} \cdot 10^{-6} = \mathbf{924.8\ MPa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{924.8 \times 10^6\ Pa}{70 \times 10^9 Pa} = \mathbf{0.0132}$$

$$\varepsilon_y = \text{Type equation here.}$$

<u>Buckling</u>

Regarding the long and short columns described in the lab manual, which are the same as the columns used in the experiment:

For the long column, the critical load P_{cr} was determined to be **4340 N** while the critical load for the short column was determined to be **5840 N.** For comparison, the theoretical values for the critical loading for each column were also calculated (Table VI) using equation (7). The theoretical loads were calculated as **2494 N** and **3642 N** for the long and short columns respectively (although, if you use the theoretical value for E of 70 GPa, the theoretical Pcr for the long and short columns become **4141 N** and **6045 N** respectively – see the Additional Discussion section).

1. Are the range of loadings described in the lab manual for the two columns appropriate to generate the Southwell plot for these columns?

No, I don't think so. The Southwell plot could have been generated with loads that were lower than those utilized in the lab manual. If you take a look at Figures 5 and 6, you'll see that the last two data points of each plot are major outliers in terms of magnitude of strain vs. strain/applied load. However, the points on the plot generated before the last two look like they fall on a very similar slope. Each beam could have been subjected to about a kN less than what the lab manual called for and we still would have found a slope that would have been enough to generate a Southwell plot showing Pcr.

Additional Discussion

<u>Elastic Modulus discrepancy</u>

The calculated experimental value from this experiment was around 40% off from the tabulated value of E = 70 GPa for Aluminum. While the experiment could have had some small errors (cyclic fatigue in the strain gauge wires, discrepancies caused by the drift of the measurements after loading, the "point" load being difficult to hold in place) I can't think that the small errors would compound to anything over a 5% experimental error, let alone 40%.

Anecdotally I found out that the same aluminum specimen has been the subject of cantilever experiments in various forms in the GTAE department for years if not decades. If that were not the case, I would question whether the specimen was indeed aluminum, and how the aluminum was manufactured. I might perform the experiment again to see if there were not an error in the measuring or reporting system our team utilized.

As it stands, I have to assume the error was in the data reduction stage, but due to the finite time resources I have been unable to find the source of the error. I recommend that this report be reviewed and returned for revision if the true experimental value of the young's modulus of the cantilever specimen is desired.

<u>Pcr discrepancies</u>

The differences between theoretical and calculated value for the long and short columns were **1846** N and **2198 N** respectively. This is a huge error. However, if I eschew using my experimentally-determined value for the elastic modulus for Aluminum, the difference between the theoretical and experimental values narrows to **199 N** and **205 N** respectively, which falls much more into the acceptable range, although is still an error of around 5%. One factor in the error could have been that the loads were predetermined to be "round number" loads (i.e., 300 lbs, 400 lbs, 500 lbs), but when I grabbed the data from the video, I was only able to find the strain at loads that were close to the targeted numbers (298 lbs, 403 lbs, etc). Instead of reporting these loads to correlate with the measured strains I grabbed from the video recording, I stuck with the original "round numbers." This would allow for a slight discrepancy in data, but I deemed this practice tolerable because Pcr would be calculated using a best fit line, which inherently rounds and averages.

Tables and Figures

Table I. Measured rosette strains ε_A, ε_B, and ε_C and inferred local strains (ε^*_x, ε^*_y, and γ^*_{xy}) for each of the applied loads.

Load (g)	Load (N)	Measured Rosette Strains (microstrains)			Inferred local strains (microstrains)		
		εA	εB	εC	ε*x	ε*y	γ*xy
200	1.962	108	157	8	108	8	198
400	3.924	215	316	17	215	17	400
600	5.886	324	473	25	324	25	597
800	7.848	431	631	34	431	34	797
1000	9.81	535	787	42	535	42	997
0	0	-8	-1	-1	-8	-1	7

Table II. Measured rosette strains ε_A, ε_B, and ε_C and inferred local strains (ε^*_x, ε^*_y, and γ^*_{xy}) for each of the applied deflections δ.

δ (in)	δ (m)	Measured Rosette Strains (microstrains)			Inferred local strains (microstrains)		
		εA	εB	εC	ε*x	ε*y	γ*xy
0.1	0.00254	106	155	9	106	9	195
0.2	0.00508	214	310	18	214	18	388
0.3	0.00762	324	465	25	324	25	581
0.4	0.01016	433	619	31	433	31	774
0.5	0.0127	543	771	37	543	37	962
0.6	0.01524	638	928	52	638	52	1166

Table III. Linear proportionality constants for ε_1 vs P and ε_1 vs δ and inferred beam elastic modulus and effective bending stiffness.

Linear Proportionality Constants		Inferred Quantities	
ε1 vs P (strain/N)	ε1 vs δ (strain/m)	Elastic modulus (Gpa)	Effective Bending Stiffness (N/m)
0.00013502	0.10215	42.2	861.2

Table IV. Measured strains for each of the applied (measured) axial loads for long column.

Long Column		
Load N	ε_a	ε_b
444.8222	-0.00003	-0.000021
1334.466	-0.000092	-0.00006
2224.111	-0.000162	-0.000092
3113.755	-0.000261	-0.000099
4003.399	-0.000751	0.000288
3558.577	-0.000434	0.000031
2668.933	-0.000231	-0.000062
2224.111	-0.00018	-0.000062
1779.289	-0.000136	-0.000049
889.6443	-0.000065	-0.000013

Table V. Measured strains for each of the applied (measured) axial loads for short column.

Short Column		
Load N	ε_a	ε_b
2224.111	-0.00012	-0.00012
3113.755	-0.00017	-0.00016
4003.399	-0.00023	-0.0002
4893.044	-0.00031	-0.00021
5782.688	-0.00047	-0.00015
5337.866	-0.0004	-0.00017
4448.222	-0.0003	-0.00018
4003.399	-0.00025	-0.00018
3558.577	-0.00022	-0.00017
2668.933	-0.00015	-0.00014

Table VI. Experimentally determined and predicted buckling loads for each column

Experimentally Determined Buckling Load (N)		Theoretical Buckling Load (N)	
Long Column	Short Column	Long Column	Short Column
4340	5840	2494	3642

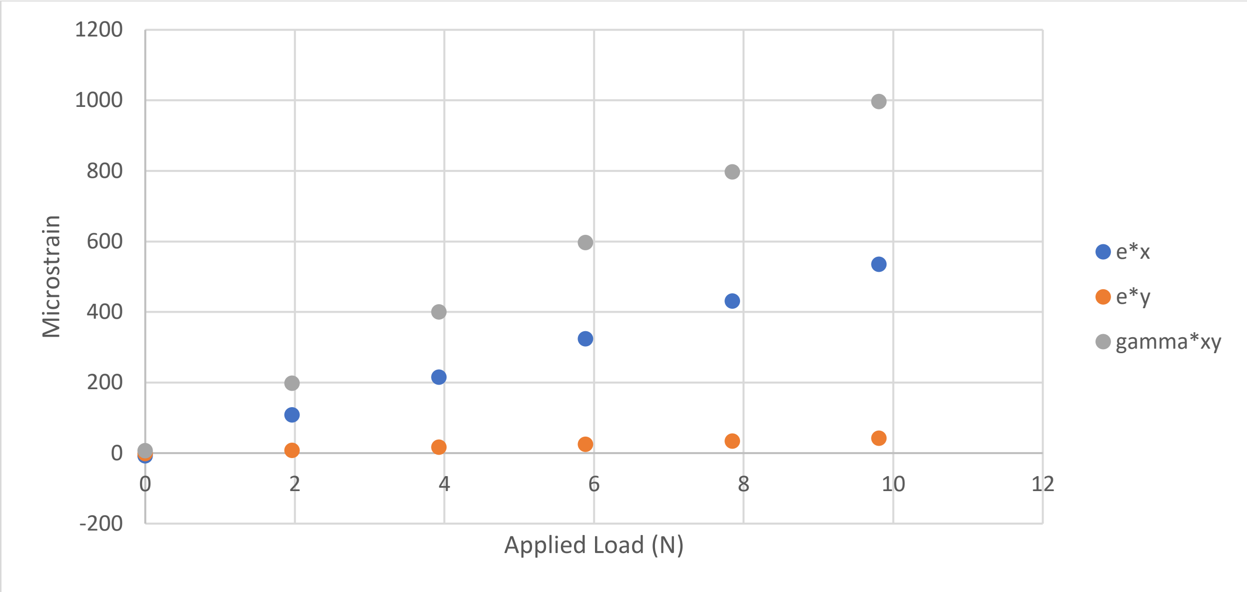

Figure 1. Combined plot of ε_x, ε_y, and γ_{xy} as a function of the applied load P.

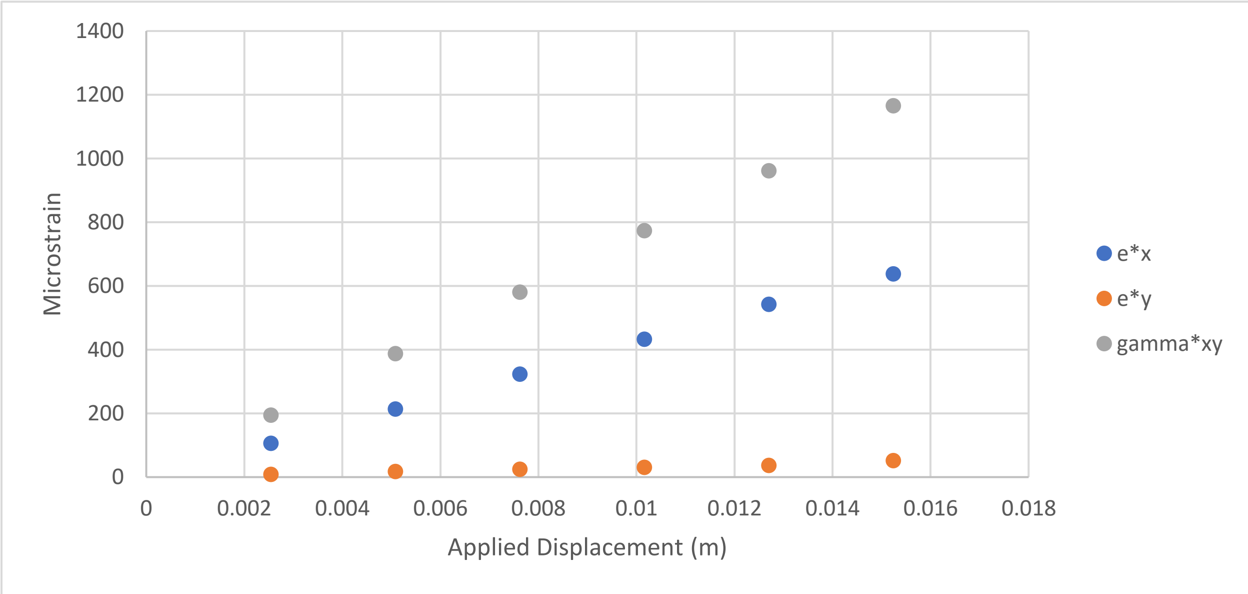

Figure 2. Combined plot of ε^*_x, ε^*_y, and γ^*_{xy} as a function of the applied deflection δ.

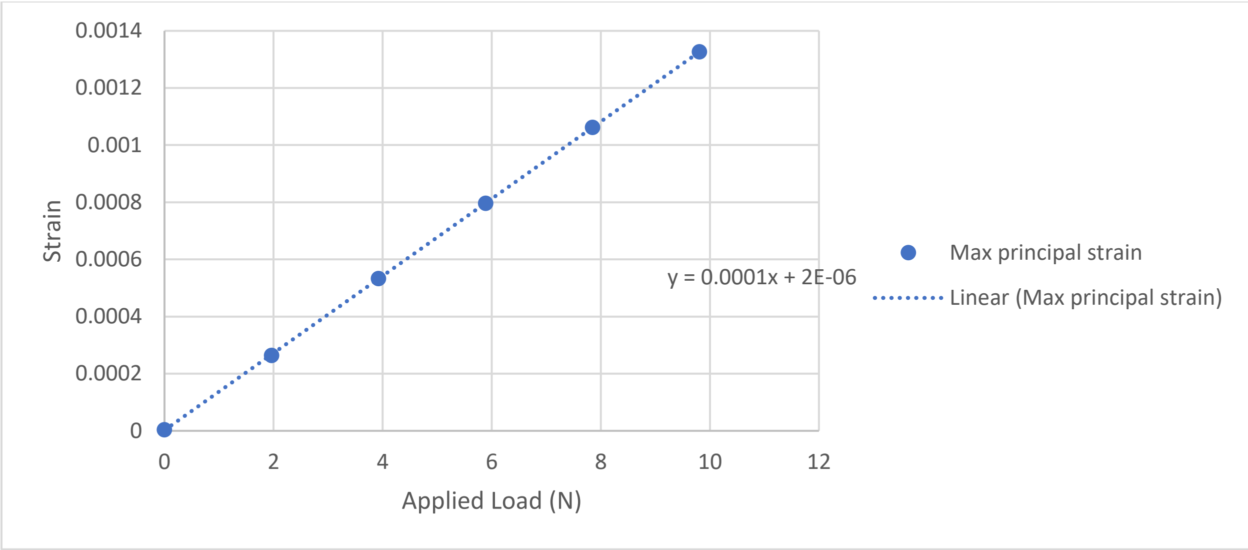

Figure 3. ε_1 as a function of applied load with best-fit line.

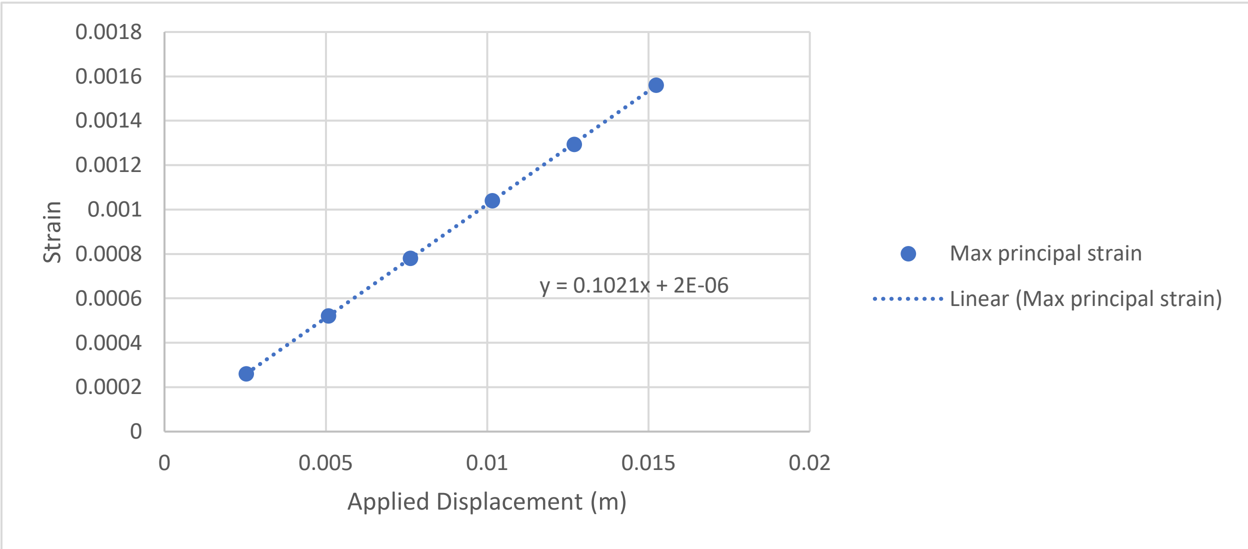

Figure 4. ε_1 as a function of δ, applied displacement, with best-fit line.

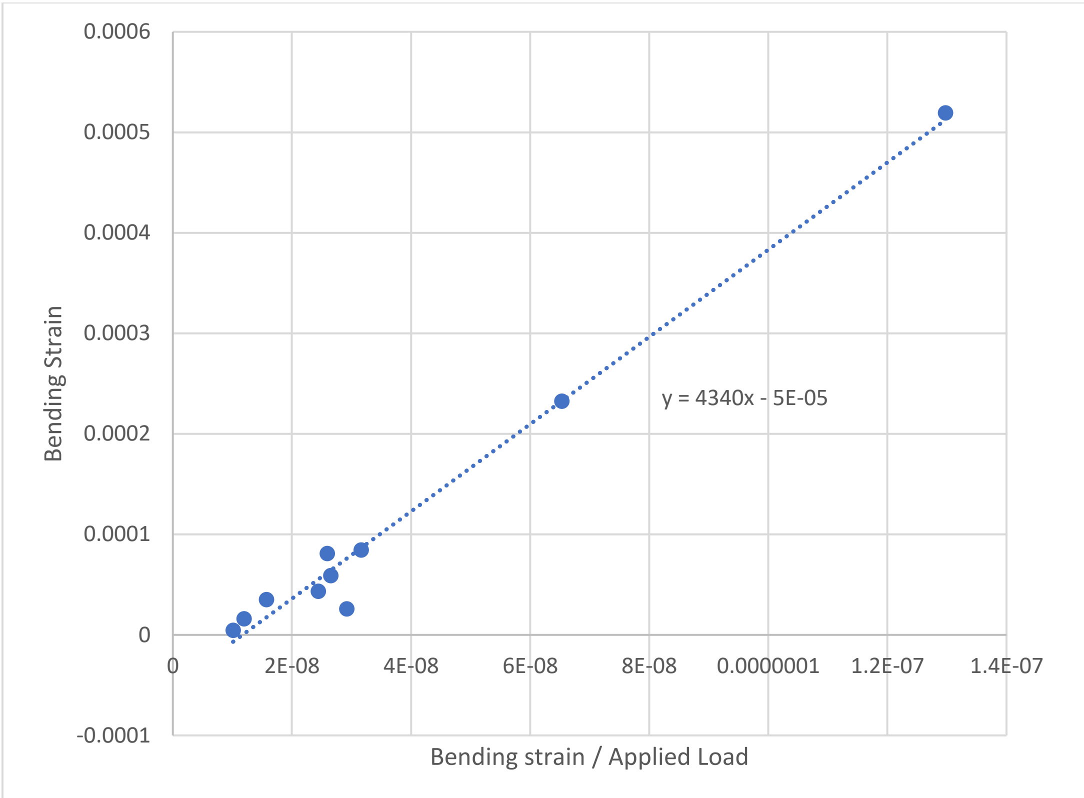

Figure 5. Southwell plot for the long column with best fit line.

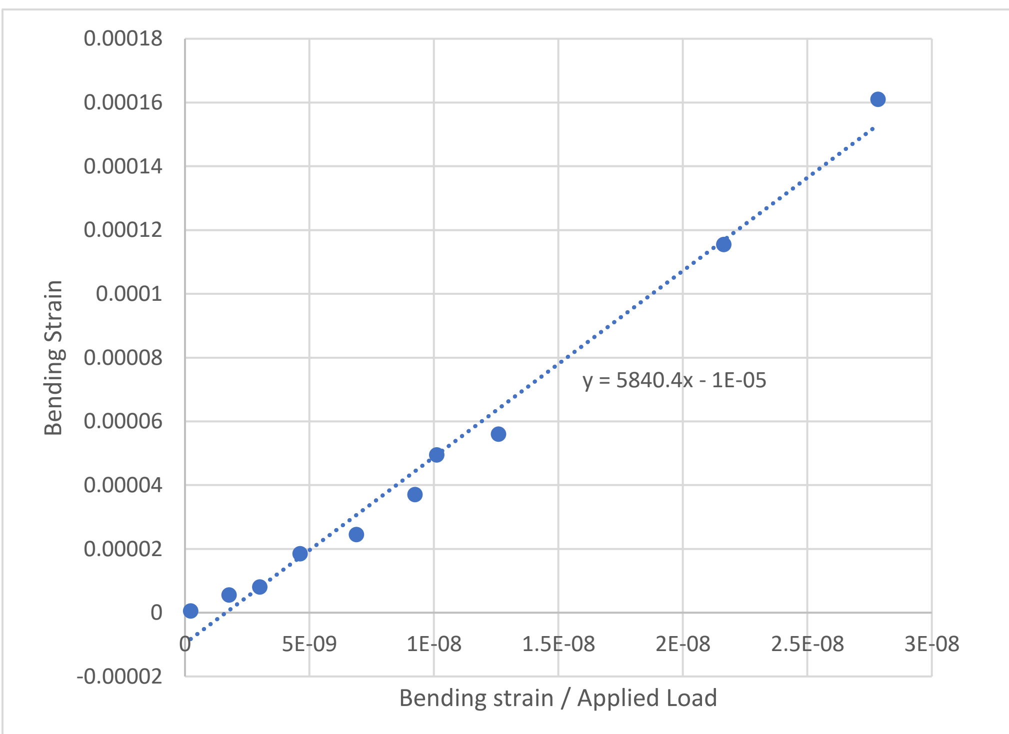

Figure 6. Southwell plot for the short column with best-fit line.